Dynamics of Cosmological Models in the Brane-world Scenario

Antonio Campos* and Carlos F. Sopuerta*

*Relativity and Cosmology Group,
Portsmouth University, Portsmouth PO1 2EG, Britain

Abstract. We present the results of a systematic investigation of the qualitative behaviour of the Friedmann-Lemaître-Robertson-Walker (FLRW) and Bianchi I and V cosmological models in Randall-Sundrum brane-world type scenarios.

Recently, Randall and Sundrum have shown that for non-factorizable geometries in five dimensions the zero-mode of the Kaluza-Klein dimensional reduction can be localized in a four-dimensional submanifold [1]. The picture of this scenario is a five-dimensional space with an embedded three-brane where matter is confined and Newtonian gravity is effectively reproduced at large distances.

Here, we summarize the qualitative behaviour of FLRW and Bianchi I and V cosmological models in this scenario (see [2] for more details). In particular, we have studied how the dynamics changes with respect to the general-relativistic case. For this purpose we have used the formulation introduced in [3]. From the Gauss-Codazzi relations the Einstein equations on the brane are modified with two additional terms. The first term is quadratic in the matter variables and the second one is the electric part of the five-dimensional Weyl tensor. In this communication we will consider the effects due to the first term. The study including both corrections has been carried out in [4]. We also assume that the matter content is described by a perfect fluid with energy density, ρ, and pressure, p, related by a linear barotropic equation of state, $p = (\gamma - 1)\rho$ with $\gamma \in [0, 2]$.

When the brane dynamics is described by a FLRW model we find five generic critical points: the flat FLRW models (F); the Milne universe (M); the de Sitter model (dS); the Einstein universe (E); and the non-general-relativistic Binétruy-Deffayet-Langlois (BDL) model (m) [5]. The dynamical character of these critical points and the structure of the state space depend on the equation of state, or in other words, on the parameter γ. This means that we have bifurcations for some values of γ, namely $\gamma = 0, \frac{1}{3}, \frac{2}{3}$. The bifurcation at $\gamma = \frac{1}{3}$ is a genuine feature of the brane world and is characterized by the appearance of an infinite number of non-general-relativistic critical points. The Einstein Universe critical point appears

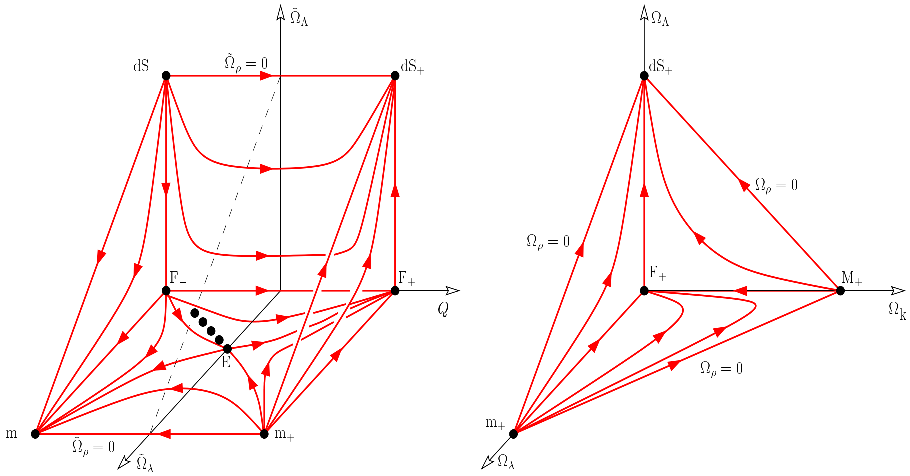

FIGURE 1. State space for the FLRW models with $\gamma \in (\frac{1}{3}, \frac{2}{3})$ and non-negative (left) and non-positive (right) spatial curvature. The variables $(\Omega_\rho, \Omega_k, \Omega_\Lambda, \Omega_\lambda)$, and their analogs with tilde, are fractional contributions of the energy density, spatial curvature, cosmological constant and brane tension, respectively, to the universe expansion [2]. Replacing Ω_k by Ω_σ (the shear contribution) and M_+ by K_+, the drawing on the right is also the state space for Bianchi I models with $\gamma \in (1, 2)$. For clarity, only trajectories on the invariant planes have been drawn. The dynamics of a general trajectory can be inferred from them. The subscript "+" ("−") refers to the expanding (contracting) character of the models. The planes $\tilde{\Omega}_\lambda = \Omega_\lambda = 0$ correspond to the state space of general relativity.

for $\gamma \geq \frac{1}{3}$, in contrast with the general-relativistic case, where it appears for $\gamma \geq \frac{2}{3}$. Actually, for $\frac{1}{3} < \gamma < 2$ we do not have an isolated critical point corresponding to the Einstein universe but a line of critical points, as can be seen in the state space shown in Figure 1. Another important feature of these scenarios is that the dynamical character of some of the points changes. For instance, the expanding and contracting flat FLRW models, which in general relativity are repeller and attractor for $\gamma > \frac{2}{3}$, are now saddle points for all values of γ. The new non-general-relativistic critical point, the BDL solution [5], describes the dynamics near the initial Big-Bang singularity and, for recollapsing models, near the Big-Crunch singularity. More precisely, the dynamical behaviour near these singularities is governed by a scale factor $a(t) = t^{1/(3\gamma)}$ which differs from the standard evolution in general-relativistic cosmology, where $a(t) = t^{2/(3\gamma)}$. Finally, the general attractor for ever expanding universes is, as in general relativity, the de Sitter model. For recollapsing universes, which now appear for $\gamma > \frac{1}{3}$, the contracting BDL model is the general attractor. However, if we only consider the invariant manifold representing general relativity, the contracting Friedmann universe is the general attractor for $\gamma > 2/3$. On the other hand, for zero cosmological constant and $\gamma < 2/3$ the expanding Friedmann universe is also an attractor.

For the homogeneous but anisotropic Bianchi I and V cosmological models, which

contain the flat and negatively curved FLRW models respectively, we find an additional critical point, namely the Kasner vacuum spacetimes (K). In the Bianchi I case the state space can be represented by the same type of drawings used for the non-positive spatial curvature sector of the FLRW evolution (see Figure 1). A representative set of diagrams for Bianchi V models is given in [2]. For Bianchi I models we have found a new bifurcation at $\gamma = 1$ and for Bianchi V models at $\gamma = \frac{1}{3}, 1$, in addition to the general relativity bifurcations at $\gamma = 0, 2$ and $\gamma = 0, \frac{2}{3}, 2$, respectively. Some of the dynamical features explained above for the FLRW are shared by the these Bianchi models. However, the most interesting point here is the possibility of studying the dynamics of anisotropy in brane-world scenarios. Specifically, we have seen [2] that, although now we can have intermediate stages in which the anisotropy grows, expanding models isotropize as it happens in general relativity. This is expected since the energy density decreases and hence, the effect of the extra dimension becomes less and less important. The situation near the Big Bang is more interesting. In the brane-world scenario anisotropy dominates only for $\gamma < 1$, whereas in general relativity dominates for all the physically relevant values of γ.

To conclude, let us summarize the main features of the dynamics of cosmological models on the brane. First, we have found new equilibrium points, the BDL models [5], representing the dynamics at very high energies, where the extra-dimension effects become dominant. Thus, we expect them to be a generic feature of the state space of more general cosmological models in the brane-world scenario. Second, the state space presents new bifurcations for some particular equations of state. Third, the dynamical character of some of the critical points changes with respect to the general-relativistic case. Finally, for models in the range $1 < \gamma \leq 2$, that is for models satisfying all the ordinary energy conditions and causality requirements, we have seen that the anisotropy is negligible near the initial singularity. This naturally leads to the questions of whether the oscillatory behaviour approaching the Big Bang predicted by general relativity is still valid in brane-world scenarios. We are currently investigating this issue by considering Bianchi IX cosmological models [6].

Acknowledgments: This work has been supported by the European Commission (contracts HPMF-CT-1999-00149 and HPMF-CT-1999-00158).

REFERENCES

1. L. Randall and R. Sundrum, Phys. Rev. Lett **83**, 4690 (1999).
2. A. Campos and C. F. Sopuerta, Phys. Rev. D **63**, 104012 (2001).
3. T. Shiromizu, K. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000).
4. A. Campos and C. F. Sopuerta, submitted to Phys. Rev. D (hep-th/0105100).
5. P. Binétruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000).
6. A. Campos and C. F. Sopuerta, *in preparation*.